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MA–1841

B. Tech. (EI) (First Semester)

EXAMINATION, 2020

MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : Attempt questions from both Sections as directed.

Section—A

(Short Answer Type Questions)

Note : Attempt any *ten* questions. Each question carries 4 marks. 10×4=40

1. Find *n*th differential coefficient of $x^3 \cos x$.

2. If :

$$u = f\left(\frac{y}{x}\right),$$

show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

3. Explain $\log(1 + x)$ by Maclaurin's theorem.

4. Prove that :

$$\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} = 1.$$

5. The function :

$$u = x^2 + y^2 + 6x + 12$$

is minimum at what value ?

6. Transform :

$$\iint f(x, y)\, dx\, dy$$

to polar coordinate.

7. Find the whole area of the curve :

$$a^2y^2 = x^3(2a - x)$$

8. Evaluate :

$$\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$$

9. If $r = |\bar{r}|$, where :

$$\bar{r} = xi + yj + zk$$

prove that :

$$\nabla r^n = nr^{n-2}\bar{r}$$

10. Determine constant a so that vector :

$$V = (x + 3y)\,i + (y - 2z)\,j + (x + az)\,k$$

is solenoidal.

11. Evaluate :

$$\int_C F.dr$$

where $F = x^2 i + y^3 j$ and curve C is $y = x^2$ in xy-plane from (0, 0) to (1, 1).

12. Prove that matrix :

$$A = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$$

is unitary.

13. Find the rank of matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -4 & 1 \\ 3 & 6 & 5 \end{bmatrix}$$

14. Solve by Cramer's rule :

$$x + 2y = 2$$
$$3x - 4y = 11.$$

15. Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} = \pi$$

Section—B

(Long Answer Type Questions)

Note : Attempt any *three* questions. Each question carries 20 marks. 3×20=60

1. Verify Stokes' theorem for :

$$F = (x^2 + y^2)i - 2xyj$$

taken round the rectangle bounded by :

$$x = \pm a$$
$$y = 0$$
$$y = b.$$

2. Find the characteristic roots and corresponding characteristic vector of :

$$A = \begin{bmatrix} 4 & 1 & -2 \\ -1 & 2 & -5 \\ 1 & 1 & -5 \end{bmatrix}$$

3. Trace the curve :

$$y^2(a^2 + x^2) = x^2(a^2 - x^2)$$

4. Find the volume of the solid surrounded by the surface :

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1.$$

5. Show that the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is $\dfrac{8abc}{3\sqrt{3}}$.

6. If :

$$u = \sin^{-1} \frac{x + y}{\sqrt{x} + \sqrt{y}}$$

prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u .$$

1

$\lambda^3 - 11\lambda^2 + 96\lambda + 24$

$\lambda^2(\lambda - 11)$ $\lambda = 6$

~~27 – 99~~

360